Name: ______________________ GTID: ______________________

- Fill out your name and Georgia Tech ID number.
- This quiz contains 3 pages. Please make sure no page is missing.
- The grading will be done on the scanned images of your test. Please write clearly and legibly.
- Answer the questions in the spaces provided. We will scan the front sides only by default. If you run out of room for an answer, continue on the back of the page and notify the TA when handing in.
- Please write detailed solutions including all steps and computations.
- The duration of the quiz is 30 minutes.

Good luck!

1. (33 points) Determine the longest interval in which the given initial value problem

$$ty'' + 3y = t, y(1) = 1, y'(1) = 2$$

is certain to have a unique twice differentiable solution. You do not need to find the solution.

The equation $ty''+3y=t$ is equivalent to $y''+\frac{3}{t}y-1=0$, whenever all the coefficients are continuous. The function $f(t)=-1$ is continuous everywhere, and the function $f(t)=3/t$ is continuous as long as $t \neq 1$. Thus possible answers are $(-\infty,0)$ and $(0,\infty)$. But the IVP is set up at $t_0=1$ and $1 \in (0,\infty)$, thus by the existence and uniqueness theorem, the answer is : $(0,+\infty)$.

2. (33 points) Find the general solution for

$$y'' - 3y' + 2y = 0.$$

We solve the characteristic equation

$$r^2 - 3r + 2 = 0$$

$$r_{1,2} = \frac{3 \pm \sqrt{9-8}}{2} = \begin{cases} 2 \\ 1 \end{cases}, \text{ thus}$$

$r_1 = 2$ and $r_2 = 1$. The roots are real and distinct, hence the general solution is:

Answer $y(t) = c_1 e^{t} + c_2 e^{2t}$,

where c_1 and c_2 are constants.

3. (34 points) Solve

$$x^2y''(x) + xy'(x) + 4y(x) = 0$$

when $x > 0$.

Consider the change of variables $x = e^t$ and $z(t) = y(e^t)$ we see that $y(x) = z(\log x)$, and

$z'(t) = e^t y'(e^t) = x y'(x)$;

$z''(t) = e^t y'(e^t) + e^{2t} y''(e^t) = x y'(x) + x^2 y''(x)$

Thus the equation is equivalent to

$$z''(t) + 4z(t) = 0 \qquad (\star)$$

The characteristic equation is

$$r^2 + 4 = 0$$

$r_1 = 2i$, $r_2 = -2i$ are the roots

Thus the solution of $(\star)$ is

$$z(t) = c_1 \cdot \cos(2t) + c_2 \sin(2t)$$

Now recall that $y(x) = z(\log x)$, thus

Answer: $\boxed{y(x) = c_1 \cos(2 \log x) + c_2 \sin(2 \log x)}$